Agent based modeling

Bottom → up

An agent-based model consists consists of a systems of agents with specified relations between them [1, 2, 3, 4, 5, 6]. Agent-based models are interesting because of their focuss on emergent phenomena: How does big systems behave as function of repeated simple interactions among relatively simple agents. Agent based models have been used to study a range of living systems, including seggregation [2], traffic jams, evacuation behaviour [4] social insect organization [8], stock market behaviour, as well as pattern formation and cellular automatons [1, 3]. Thinking of emergence in terms of actions of individuals is also behind important papers on information assymetry in markets [7] as well on possible explanations of scale free distributions of wealth as well as usage of words [9].

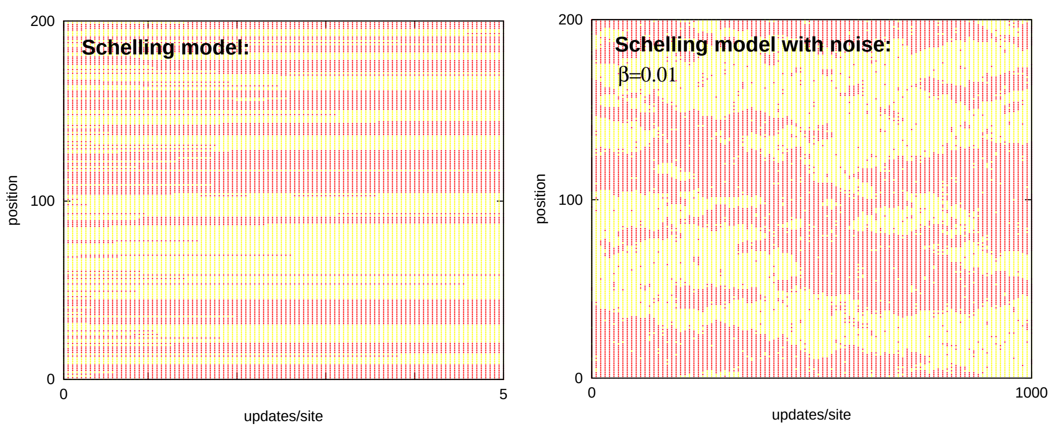


Figure 1: *Simulation of schelling model, demonstrating seggregation of agnets with two colors "red" or "yellow". At each step one agent is selcted and attempt to replace position with another randomly selected agent in the system. The attempted move is accepted if the number of opposite color agents within the nearest 4 neighbors of the selected agent is reduced. Left panel show how the agent seggregate within a few updates per aget in the system but then freezes as no further movements are possible. Right panel show that accepting suboptimal moves with some small proability* $\beta = 0.01$ *opens for segregation into larger homogeneous regions.*

Sometimes agent based thinking can inspire analytical understanding of a particular phenomenon, but this is not a requirement for being usefull. Rather, focussing on the basic units and their interactions, one may obtain a valid description of a system where a analytically solvable model would fail. Defining relevant space of interactions between the agents is the key and the art of any succesfull model.

Perhaps the earliest simple agent based model was the one proposed by T.C. Schelling [2] in 1969 to describe the apparent seggregation in white and black neighborhood, see Fig. 1. A seggregation model which can possible also be of relevance in seggregation of various cells types into tissues using the Differential adhesion hypothesis [11, 12].

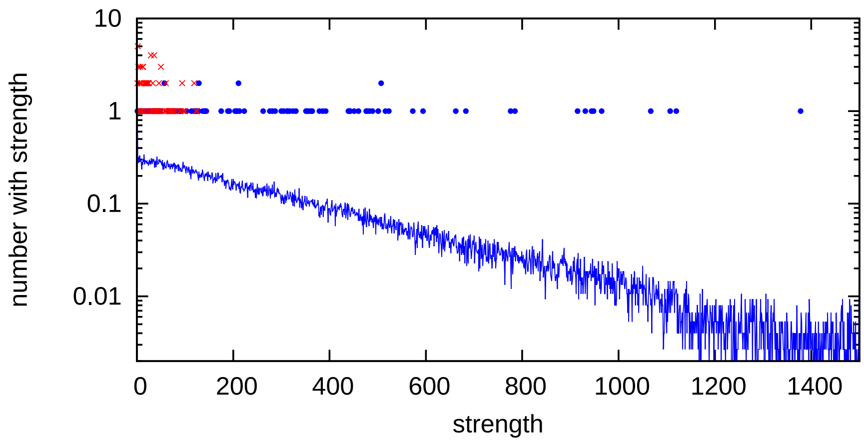


Figure 2: *Steady state distribution of "strength" distribution from a long simulation of a social hierarchy model. In the model there is N=100 agents that each is have strengths* $s \geq 1$. *At time zero* $s_i = 1$ *for all agents* i. *At each timestep two agents* i *and* j *are selected, and* i *is assigned to be winner with probability* $s_i/(s_i + s_j)$, *and otherwize* j *is the winner. The winner increase its strength* $s \rightarrow s + 1$, *whereas the loser decrease its* s *1 unit provided that it remain* ≥ 1. *After N such updates, one agent is reset to strength* $s = 1$. *Red dots represent one early distruibution, whreas blue dot represent one late distribution of strengths.*

Another example is a minimal version of a hierarchy model, simulating a positive feedback between winning and future chance of winning [14, 15]. As in the above cited references, we here let agents fight for status, and increases in this status provided that they win encounters over competitors. Even with simple rules, the system develop towards a steady state characterized by a hierarchy with few in the top and exponentially many more agents in the bottom of the hierarchy, see in Fig. 2. Notably, as there is a net increase in fighting strength with the simple fighting rule of Fig. 2, random agents are also assumed to die and be replaced by new agents at some slow rate. It is this random 'killing" that secure a steady state distribution of the strength distribution. A steady state distribution where these is few in the top and many more at each lower level.

- **Lesson 1:** Social structure can emerge as consequence of many itterations of simple update rules between pairs of agents.

Questions:
1) Simulate the Schelling model in 2-dimension, taking only into account nearest neighbors (left-right and up and down). Allow moves where the agen keeps exactly the same number of opposing type neighbors.
2) Simulate a Schelling like model with 3 colors, all of which want to minimize exposure to each other. Also simulate a variant where only 2 colors mutually want to minimize number of neighbors of opposing color, whereas one color moves randomly.
2) Simulate the hierarchy model in the text, wit the constraint that agents walks randomly on a 1-d line (let agents at neighbor position switch positions each time they have a hierarchy battle).

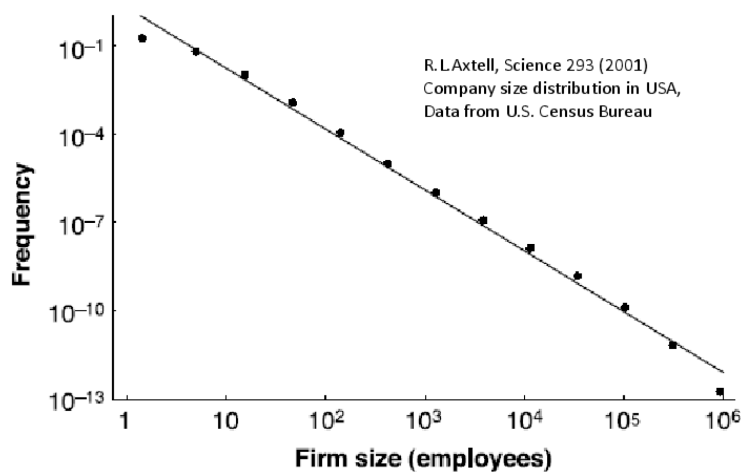


Figure 3: *Company size distribution in USA, exhibiting a scale free behaviour with exponent -2 (data from R.L. Axtel, 2001).*

Emergence of an unfair society from a fair game

Value and wealth are exposed to multiple random factors, that vary hugely across society [18] and unpredictably in time [17]. In principle, wealth opens up for investment, and thus for generating more wealth, in a self-amplifying process. However, investments are rarely profitable immediately, and potential gain is potentially deteriorated by random events until the investment has matured and its eventual profit can be realized.

The skewness of wealth distributions [18, 19] have inspired a number of ad-hoc modeling approaches, emphasizing explicitly the advantage of the rich in dealing with the poor while redistributing wealth [20, 9]. Most notably, the preferential accumulation [9] of wealth naturally generates scale-free wealth distributions with fortune f occurring with probability $\propto 1/f^{\gamma}$ with $\gamma > 2$.

An alternative approach to obtain skewed wealth distributions has its roots in multiplicative random processes [37, 22], where in particular [23, 24] suggested that skewed wealth distributions may reflect a stochastic multiplicative process with a drift.

Following Bornholdt & Sneppen (preprint, 2012) we here introduces a simple model of wealth accumulation, with similarities to a multiplicative random process. It is a non-interactive agnet model where wealth is not generated by competition, but by simple gambling assuming that any investment is a fair gamble. That is, any investment is defined to have a 50/50% chance to succeed, and if it succeeds the gain exactly equals to the loss associated to its potential failure.

The model aims at drawing an anecdotal sketch of the speculation mode, or casino mode, of real markets. Investments are possible but their return is, on average, exactly zero. The quit-or-double game inevitably leads to bankruptcy. We call an agent "bankrupt" once he reaches zero wealth and thus is not able to continue the game. However, new agents may constantly enter the system with some small fortune, a minimal unit we here set to $f = 1$. The wealth distribution of the quit-or-double game is obtained by iterating fortunes $f \to 2 \times f$, respectively $f \to 1$ with equal probability. The $f \to 1$ correspond to one agent losing everything, being replaced by a new agent that enters the game with $f = 1$. The probability to reach fortune $f = 2^j$ or more is the probability to with at least j consequetive games, i.e. $(1/2)^j$. Thus $P(> f) = 1/f$, equivalent to the famous Zipf distribution for distribution of word frequencies [25]. The corresponding probability to have a fortune between f and $f + df$ is $p(f) = -dP/df = 1/f^2$. This distribution, fits perfectly the real distribution of company sizes in USA, see Fig. 3.

Remarkably, the scaling of the resulting wealth distribution $p(f) = 1/f^2$ is marginally fair, in the sense that the fraction of wealth accumulated at "the rich" is sizable but not dominating. To understand this, consider a wealth distribution with a scaling of the form $p(f) \propto 1/f^{\gamma}$ with some $\gamma > 1$. For this distribution the amount of money above a threshold T is

$$
\begin{aligned}
F(> W) &= \frac{\int_T^{Max} f df/f^{\gamma}}{\int_1^{Max} f df/f^{\gamma}} && (1)\\
&= 1 - Log(T)/log(Max) \;\; for \;\; \gamma = 2 && (2)\\
&\approx 1 - (T/Max)^{2-\gamma} \;\; for \;\; \gamma < 2 && (3)
\end{aligned}
$$

where Max represents an upper maximum that is set by system constraints.

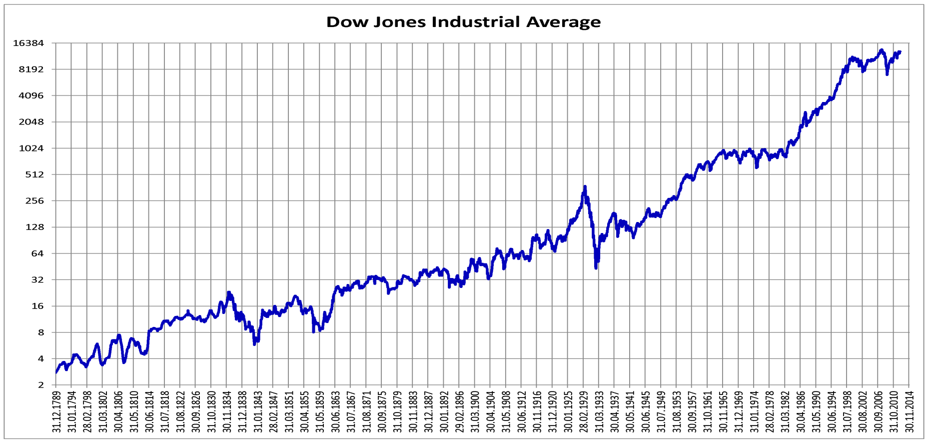


Figure 4: *Dow Jones, an index following the average of the major shares in USA. The index increase with about a factor 4000. For comparison, the US public depth changed from* $\sim 10^8\$$ *in the period 1800-1850 to* $\sim 5 \times 10^{12}\$$ *in year 2000.*

Therefore, for $\gamma < 2$ a major part of the total wealth will be accumulated at large f, say $f > T = Max/2$, whereas only a smaller fraction will be accumulated at $f > Max/2$ when $\gamma = 2$. For $\gamma > 2$ the fraction of wealth accumulated among the wealthiest (large f) is very small.

- **Lesson 2:** Scale invariant distributions of fortunes may emerge from rules that do not favour the richest.

Questions:
1) Simulate the quit-and-double game from the text. Plot the resulting wealth distribution in a society with 1000 agents. What is survival time distribution of companies?
2) Simulate a quit-and-double game where each agent is only allowed to play with one unit at a time. Plot the resulting wealth distribution in a society with 1000 agents. What is survival time of companies in this game?

Time series of stocks

Fig. 4 show a stock market index during 200 year period. The index is calculated as the average of many shares, and should thus in principle be much less variable than individual shares. In spite of this, there is indeed wild fluctuations, with occational collapses where the overall walue of all stocks drops by a factor 10 over a relatively short period. In fact, when one inspect stock markets across the world, then nearly all stock markets have had about 1 factor 10 reduction one time during the last century. Value is dynamic.

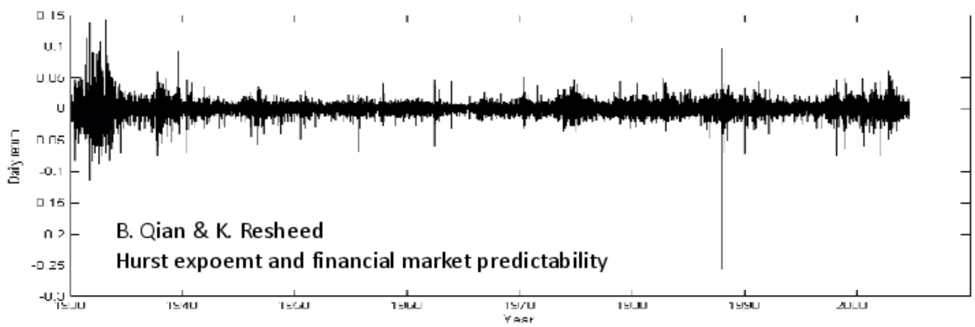


Figure 2.2. Dow-Jones daily return from 1/2/1930 to 5/14/2004

Figure 5: *Daily returns,* $(S(t) - S(t-1))/S(t-1)$*, for Dow Jones stock market index. Fluctuations are correlated: When variations on one day is large, then it most likely is large again next day. The direction of these fluctuations are uncorrelated! The size distribution of short time returns is further analyzed in subsequent figure.*

To first approximation the market exhibit a biased random walk. Or more precisely, detrending for the overall increase due to general growth of the economy/inflation, the $log(price)$ follow a random walk. The random walk hypothesis was firts put forward more than a century ago by Bachelier [17], and has been recently supported by analyzing price fluctuations $W(t)$ as funnction of time:

$$W^2(t) = \langle (log(S) - \langle log(S) \rangle_t)^2 \rangle_t \tag{4}$$

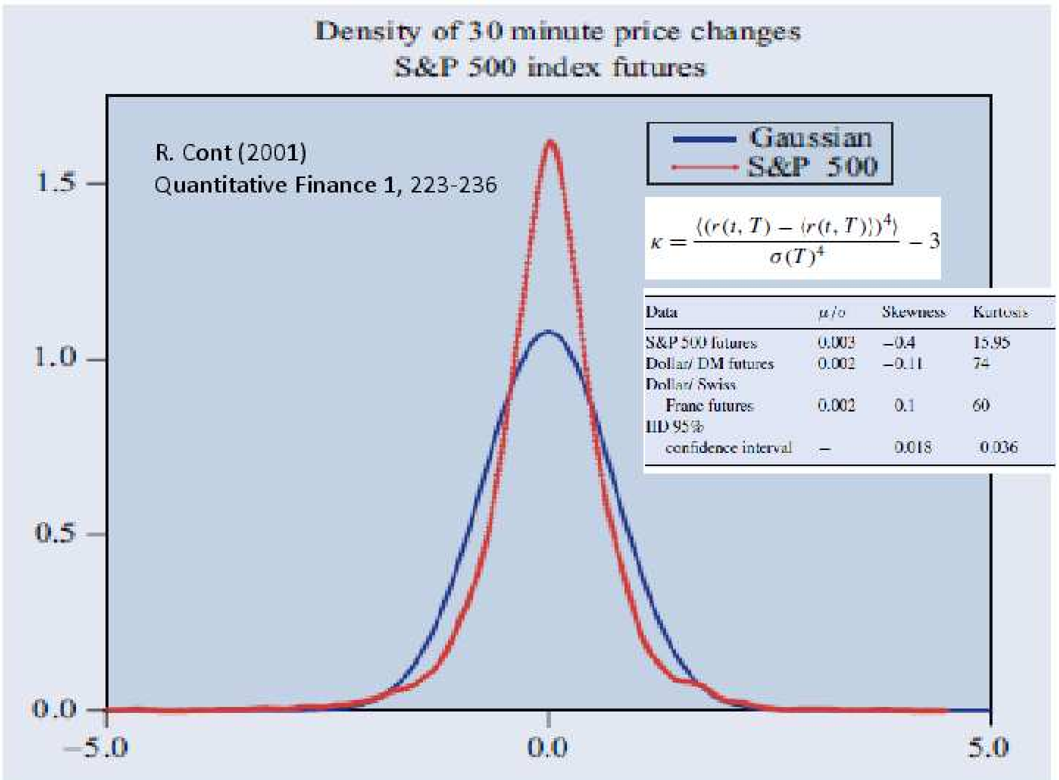
where the average is done over all time intervals of length t in the available timeseries. For a random walk $W(t) \propto t^{0.5}$, whereas most stock markets show $W(t) \propto t^{0.55 \rightarrow 0.65}$ with the lowest values of the Hurst exponent for the oldest markets. Notice that one can define the Hurst exponent in terms of both the variance of prices over a time interval with length t, or instead just define it in terms of the variation after a time interval t. In both cases it involves sampling a lot of different starting points!

Remarkably, the correlation between past and future may be related to the Hurst exponent H, defined by

$$\langle (s(t+x) - s(x))^2 \rangle_x \propto t^{2H} \tag{5}$$

where the squared price variation is averaged over all starting points x on the time-series. After some manipulation this gives [43, 44]

$$\frac{\langle -\Delta s(-t) \cdot \Delta s(t) \rangle}{\langle (\Delta s(t))^2 \rangle} = 2^{2H-1} - 1 \tag{6}$$



Data	μ/σ	Skewness	Kurtosis
S&P 500 futures	0.003	−0.4	15.95
Dollar/ DM futures	0.002	−0.11	74
Dollar/ Swiss Franc futures	0.002	0.1	60
IID 95% confidence interval	–	0.018	0.036

Figure 6: *Distribution of short timescale fluctuations exhibit fat tails. The red and blue curve have same variance, but very different Kurtosis. Kurtosis quantify the 4'th moment, normalized by second moment squared. It is more sensitive to tails in distribution that second moment, and would thus be divergent when $p(tail) \propto 1/\Delta s^{\gamma}$, with $\gamma \leq 5$ (prove that). Kurtosis vary substantially between markets, reflecting a near divergent distribution of the fouth moment of the volatility.*

Thus a ordinary random walk with $H = 1/2$ have $C = 0$, whereas a $H > 1/2$ walk imply that the past price difference $\Delta s(-t) = s(0) - s(-t)$ is most likely maintained for $\Delta s(t) = s(t) - s(0)$. Thus in the $H > 1/2$ case, a winning strategy is to "bet" on the trend: Buy when it is bull market, and sell when it becomes bear market [44]. Thus for $H > 1/2$ one should:

$$Buy\ \ at\ \ x\ \ if\ \ s(x-t)\ \ <\ \ s(x) \tag{7}$$
$$Sell\ \ at\ \ x\ \ if\ \ s(x+t)\ \ >\ \ s(x) \tag{8}$$

whereas this strategy should be reversed in a $H < 0.5$ market, see Fig. 7. Noticably, electricity markets have $H = 0.40$ [45, 46].

- **Lesson 3:** Markets are nearly random walks, but also exhibit correlations that may well reflect crowd behaviour.

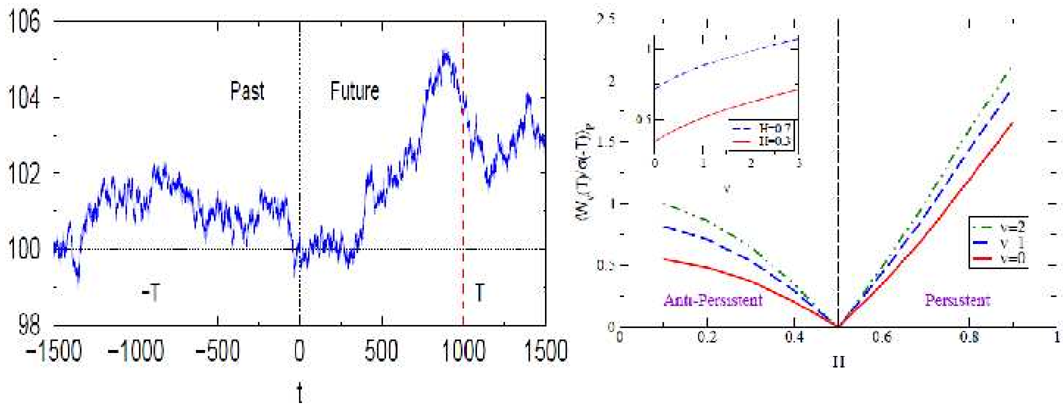


Figure 7: *Left: Example of a timeseries with Hurst exponent $H = 0.40$, generated by wavelet method (not pensum). Right panel examine average return of investment as function of H where one buy according to trend [44]. The red curves show profit when one buy on way up, and sell on way down in $H > 0.5$ markets, and oppositely in $H < 0.5$ markets. The two other curves invest proportional to size of past price change $\nu = 1$, respectively to this channge squared $\nu = 2$. Thus weighting the trend pays even more off. All returns are measured in units of spread of volatility on the considered time interval, and the curves in fact scale proportionally to this as horizon T for investment inncreases.*

Questions:

1) Simulate a walk where a price moves one step up or one step down at each timestep. Let the propability to continue in same direction as previous step be $p = 0.75$. Investigate Hurst exponent for this walk numerically.

2) Formal prof of eq. 6. Consider variation around some time point x: $\Delta_x s(t) = s(t+x) - s(x)$ and $\Delta_x s(-t) = s(x-t) - s(x)$.

Denominator $\langle \Delta^2 s(t)\rangle_x = \langle s^2(x+t) + s^2(x) - 2s(x)s(x+t)\rangle_x = 2(\langle s^2(x)\rangle - \langle s(x)s(x+t)\rangle_x) = f(t)$ where we use that averaging over all starting time points x makes $\langle s(x+t)^2\rangle_x$ and $\langle s^2(x)\rangle_x$ equal. In the end remember that the denominator is a varaince, and thus have to to scale as $f(t) \propto t^{2H}$.

Similarly the nominator $\langle -\Delta s(-t)\Delta s(t)\rangle_x = \langle -(s(x-t) - s(x))(s(x+t) - s(x))\rangle_x =$
$-\langle s(x-t)\cdot s(x+t)\rangle_x + \langle s(x-t)\cdot s(x)\rangle_x + \langle s(x)\cdot s(x+t)\rangle_x - \langle s^2(x)\rangle_x =$
$-\langle s(x)\cdot s(x+2t)\rangle_x - \langle s^2(x)\rangle_x + 2\langle s(x)\cdot s(x+t)\rangle_x - 2\langle s^2(x)\rangle_x = \frac{1}{2}f(2t) - f(t)$.
Insert $f(t) = const \times t^{2H}$ to prove eq. 6.

3) Plot eq. 3 as function of H, as well as profit of a standard strategy in terms of standard variation as function of H.

4) Interprete the correlation function eq. 6 in terms of probabilties in the case where deviations from time $-t$ to 0 is +1, whereas deviation from time 0 to t is either +1 or -1.

Fear-Factor model

To explore economic time series we here focuss on one measure, called inverse statistics [38, 39]. In turbulence where one measure velocity differences, the inverse statistics measure distribution of times/lengths until next large evlocity fluctation. Thus the inverse statistics focuss attention on the laminar regions of the fluid [38]. In economics this measure is associated to the time it takes before one obtain a given return on an investment. This will take a long time when stocks are calm, or when fluctuation is the opposing direction of what one wait for.

Let $S(t)$ denote the asset price at time t. The logarithmic return at time t, calculated over a time interval Δt, is defined as $r_{\Delta t}(t) = s(t + \Delta t)s(t)$, where $s(t) = lnS(t)$. We consider a situation in which an investor aims at a given return level, ρ, that may be positive (being long on the market) or negative (being short on the market). If the investment is made at time t the inverse statistics, also known as the investment horizon, is defined as the shortest time interval $\tau(t) = \Delta t$ fulfilling the inequality $r_{Deltat}(t)\rho$ when $\rho \geq 0$. For losses $\rho < 0$ one similarly define first time where $r_{\Delta t}(t) \leq \rho$. The inverse statistics histogram, or in economics, the investment horizon distribution, $p(\tau_p)$, is the distribution of all available waiting times $\Delta(t)$ obtained by moving through time t of the available time series.

The data set used is the daily close of the DJIA covering its entire history from 1896 till today. Fig. 8 depicts the empirical inverse statistics histograms the investment horizon distribution for (logarithmic) return levels of $\rho = 0.05$ (open blue circles) and $\rho = 0.05$ (open red squares). The histograms possess well defined and pronounced maxima, the optimal investment horizons, followed by long $1/t^{3/2}$ power-law tails.

Remarkably, the optimal investment horizons with equivalent magnitude of return level, but opposite signs, are different. Thus the market as a whole, monitored by the DJIA, exhibits a fundamental gain-loss asymmetry. As mentioned above other indices, such as SP500 and NASDAQ, also show this asymmetry, while, for instance, foreign exchange data do not.

It is even more surprising that a similar well-pronounced asymmetry is not found for any of the individual stocks constituting the DJIA. This can be observed from the insert of Figure, which shows the results of applying the same procedure, individually, to these stocks, and subsequently averaging to improve statistics. So the question is, why does the index exhibit a pronounced asymmetry, while the individual stocks do not? This question is addressed by the fear-factor model introduced below [42].

The main idea is the presence of occasional, short periods of dropping stock prices synchronized between all N stocks contained in the stock index. In essence these collective drops are the cause (in the model) of the asymmetry

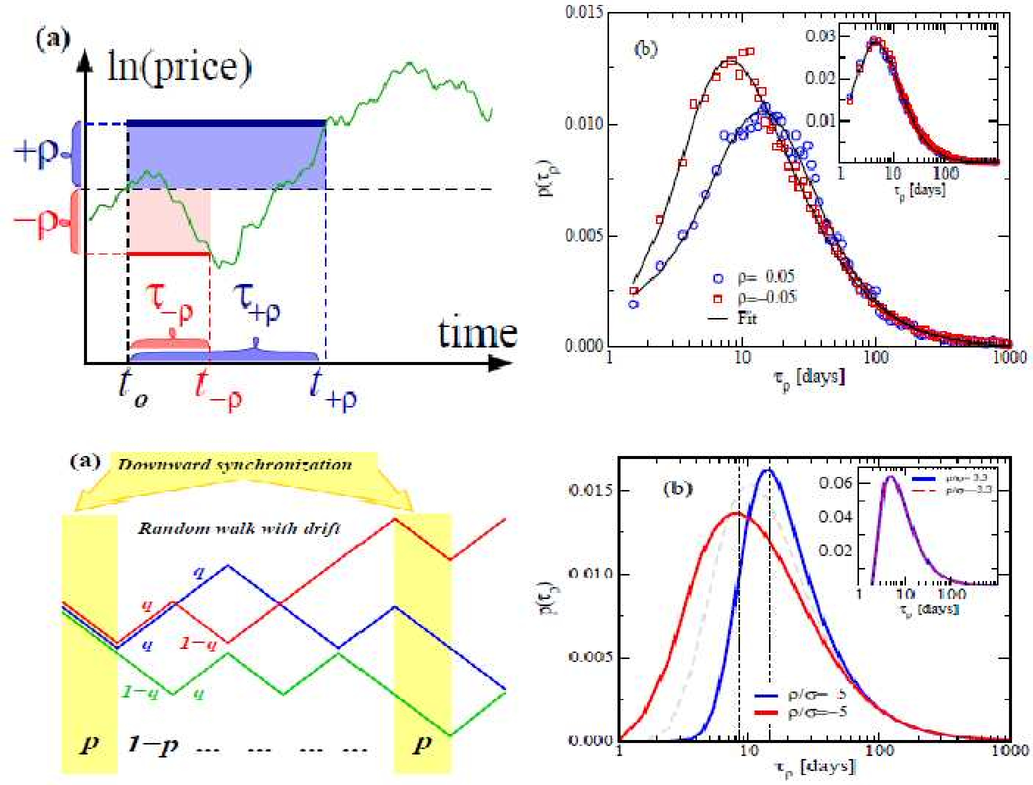


Figure 8: *Upper two panels show definition of strike price, and the distribution as measured from detrended Dow-Jones index The blue curves show number of days when price first exceed current price with 5%, the red when it first is 5%below current price (notice that insert showing corresponding strike price distributions for individual companies). Lower panels define model and show predicted strike-price distributions.*

in the index.

Is is assumed that the stochastic processes of the stocks are all equivalent and consistent with a geometrical Brownian motion This implies that the logarithm of the stock prices, $s_i(t) = lnS_i(t)$, follow standard, unbiased, random walks

$$s_i(t+1) = s_i(t) + \epsilon_i(t), \quad i = 1, ..., N \tag{9}$$

where $\delta > 0$ denotes the common fixed log-price increment (by assumption), and $\epsilon_i(t) = \pm 1$ is a random time-dependent direction variable. At certain time steps, chosen randomly with fear factor probability p, all stocks synchronize a collective draw down ($\epsilon_i = 1$). For the remaining time steps, the different stocks move independently of one another. To assure that the overall dynamics of every stock is behaving equivalent to a geometric Brownian motion the typical movement need a slight bias. Lets q be chance to move up ($\epsilon = +1$) is calm periods, and $1-q$ the probability to move down ($\epsilon = -1$). If probabilit to have collective fear and syncroneous downward move is p the probability to move up for one company is $(1-p)\cdot q$ whereas the probability to move down is $p + (1-p)\cdot(1-q)$. Neutral walk demand that these probabilities have to be equal:

$$(1-p)\cdot q = p + (1-p)\cdot(1-q) \tag{10}$$

fixing q in terms of the probability for overall fear:

$$q = \frac{1}{2\cdot(1-p)} \tag{11}$$

$q > 1/2$ is a compensating drift that governs the non-synchronized periods. From the price realizations of the N single stocks, one may construct the corresponding price-weighted index, like in the DJIA, according to

$$I(t) = \frac{1}{N}\sum_{i=1}^{N} S_i(t) \tag{12}$$

and investigate inverse statistics for this. This is done in Fig. 8. Overall result: DJIA is reproduced with one collective fear occuring with probability $p = 0.05$ per day, corresponding to one panick event per month or so. The other parameter is $\rho = 5\cdot\sigma$, where σ is standard deviation of of volatility of index and we use an index of $N = 30$ shares. For DJIA $\sigma = 1\%$.

We conclude that the asymmetric synchronous market model captures basic characteristic properties of the day-to-day variations in stock markets. The agreement between the empirically observed data here exemplified by the DJIA index and the parallel results obtained for the model gives credibility to the point that the presence of a fear-factor is a fundamental social ingredient in the dynamics of the overall market.

- **Lesson 4:** Crowd bahavior and panick on even the relative small scale of a once in a month event can be seen through use of inverse statistics.

Questions:

1) Consider the fear factor model with 10 stocks that moves one step up or down, all starting at 1000. With probability $p = 0.05$ all stocks moves down simultaneously. What should probability for other up and down movements be to mak individual stocks a random walk? Simulate the system and plot the time series for the average stock price.

Modeling a market of Products and Attention

To emphasize the "'psychogical" aspects of markets we here consider a stylized market where the cooperative emergence of "value" is associated to a positive feedback between perception of needs and demand. [40, 41, 42]

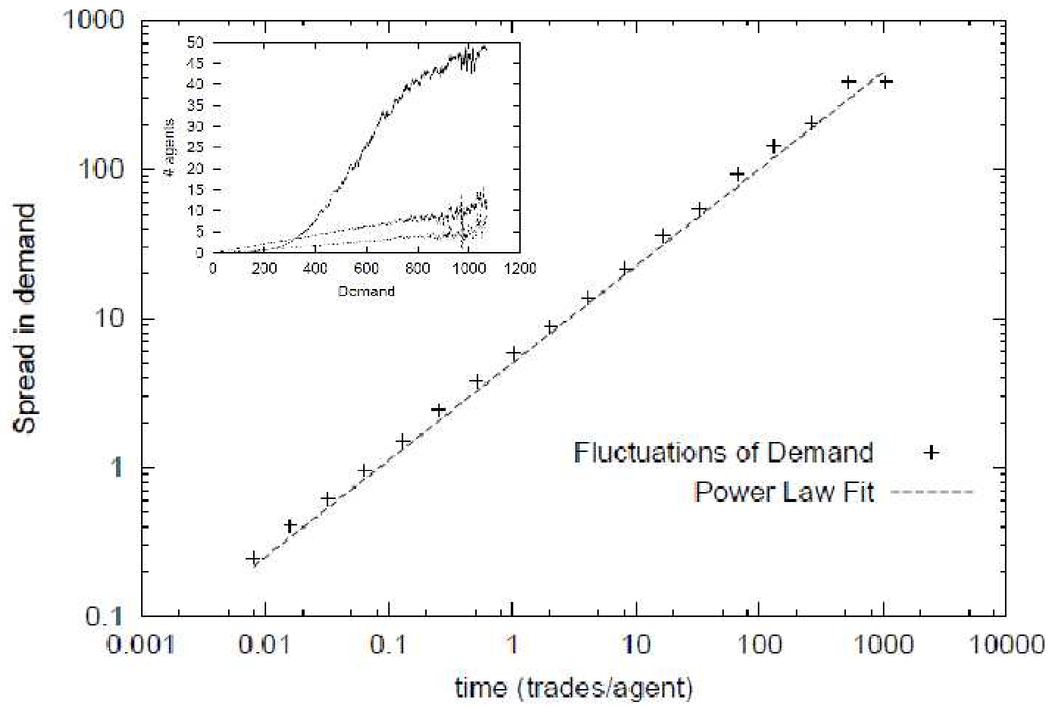


Figure 9: *Long time attention-variations in the fashion-model. The power law fit to* $W \propto t^H$ *is scaling with Hurst exponent* $H = 0.65$. *The parameters for this simulation was* $N_{ag} = 500$ *agents trading* $N_{pr} = 500$ *different products, which each comes in 1000 copies. The meory for agents was put to* $T_{mem} = 1000$ *updates. The insert refer to a 10-times smaller system emphasizing the interllay between the total demand of a particular product, and the number of agents that demand the product as per their greed (memory). The lower curves show how much the agents that actually need the product.*

The market we consider consists of N_{ag} agents and N_{pr} different products. Initially we give N_{unit} units of the products to each agent. The number N_{unit} is fixed, but the products are chosen at random, so the individuals are not in exactly the same situation. At each time-step we select two agents at random and let them attempt to perform a trade between each other. In order to perform a trade, each agent presents the other a list with the goods he is interested to obtain.

The trade start by comparing the list of goods that each agent lacks and therefore would like to get from the other agent in exchange for goods it has in stock. Therefore, the model first considered the simple **need** based exchange procedure: when each of the agents had products that the other needed, then one of these products, chosen at random, was exchanged. In case such a need based exchange were not possible they considered the **greed** exchange procedure: one or both of the agents would accept goods which they do not lacked, but considered useful for future exchanges. The chance for an agent i to accept a product j is $p_{ij} = T_{ij}/N_{mem}$ where T_{ij} is the amount of times product j appears in agent i's memory, a memory that have total size N_{mem}.

After trading, each agent update his memory list by replacing one random of hist memory slots with the memory of one of the products that his opponents showed interested in. This last step makes products that are needed, more wanted, and further spread the desire for products that some agents wants.

Fig. 9 examine statistics of the demand for one product j, $D_{ij} = \sum_i T_{ij}$, examining how much a given product fills in all agents $i = 1, 2, ...N_{ag}$ memory. This attention represent the value of a product. The fluctuations in demand is persistent, quantified by Hurst exponent $H > 0.5$. If many people think its valuable, then it is valuable, giving a positive feedback. A feedback that makes value of value, proved some seed from need. [1)].

- **Lesson 5:** Attention opens for positive feedback, which with some noise can cause large persistant fluctuations.

Language spreading & Information sorting

It is generally understood in historical linguistics that geolinguistic diffusion, the process by which linguistic features spread geographically from one dialect or language to another, plays a central role in the evolution of languages[47]. Origins of linguistic changes are plentiful where societal changes and movements are of pivotal importance. The dynamics and causes of linguistic change provide therefore important clues to the historical developments and interplay between societies [**?**].

It has long been observed that linguistic features, just like innovations, spread outward from an originating centre. Spatial patterns are, however, rather ambiguous. A beautiful example is the geographical distribution of the word 'snail' in Japan. In a celebrated work in 1927 *K. Yanagita* [49] found that ancient forms of the word still existed on the southern and northern parts

[1]Notice that we in principle have not described the model fully, as it as defined above will not give a steady state. That is, as time goes, products will occationally be distributed soall agents have at least one, and simultaneously such that no agents have the product in its memory. When this happens the product is forgotten, and effectively dissapear from the market. To prevent this from happening, one can put in a small probability p that individual products dissapear, and at the same time let agents produce random products to sustain the overall number of goods in the population. The result shown in Fig. 9 in fact refer to such a simulation.

of the country but not in the middle. He concluded, using his wave theory, that this reflected the strong influence that Kyoto, Japan's old capital.

Following *Lizana et al.* [50] we now consider the dynamics of culture spreading around strong pulsating culture centres. As a proxy for the spreading of cultural traits we use the spreading of words where the key feature of our model is that new words are more prone to adopted than old. As a special case we study word spreading in Japan which serve as a good base case of a single strong centre thriving of ideas which spread over the country.

Figure (a) shows the geographical distribution of swear words over Japan which clearly bears features of the wave theory. There are about 20 words present in the map where the overall trend is that old words are found far away from Kyoto and new words close by. The drawn circles show their centre of mass distribution with respect to the absolute distance from Kyoto. The data shows the gap between to adjacent circles are not uniform but grow with increasing distance away from Kyoto. Also, the speed of swearword propagation has been estimated to be v_{word} = 1 km/year (0.5-2 km/year) meaning that words in the northern and southern parts of Japan are about 500 years old.

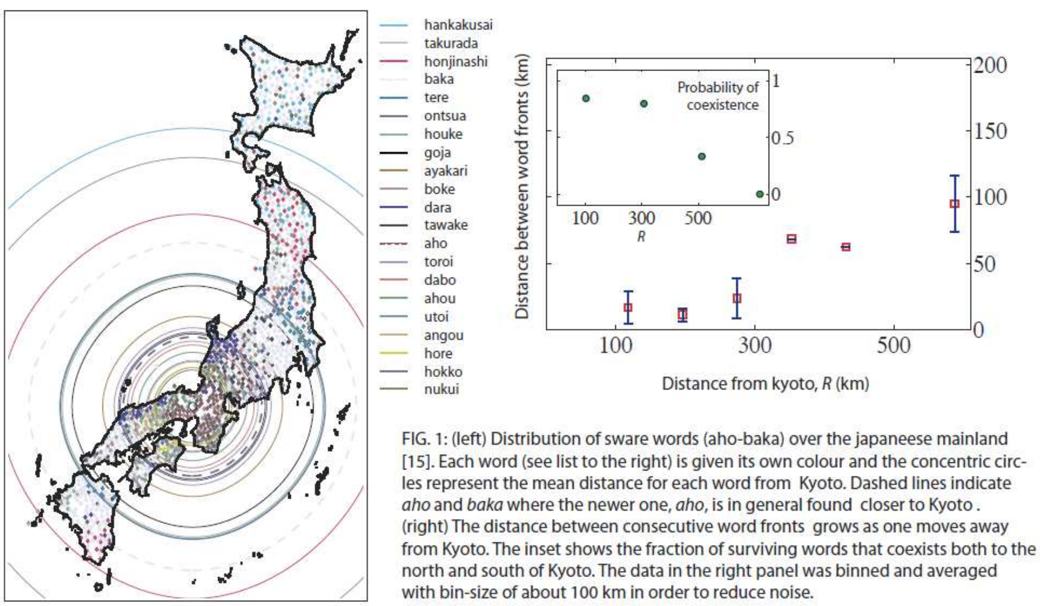


FIG. 1: (left) Distribution of sware words (aho-baka) over the japaneese mainland [15]. Each word (see list to the right) is given its own colour and the concentric circles represent the mean distance for each word from Kyoto. Dashed lines indicate *aho* and *baka* where the newer one, *aho*, is in general found closer to Kyoto. (right) The distance between consecutive word fronts grows as one moves away from Kyoto. The inset shows the fraction of surviving words that coexists both to the north and south of Kyoto. The data in the right panel was binned and averaged with bin-size of about 100 km in order to reduce noise.

Our model is defined on a two dimensional lattice on which words, after being coined in the culture centre, spread. In order to capture the ongoing adoption and subsequent communication of new words originating with a given frequency f_{word} from the centre, at each time step a word is replicated and passed on to a neighbouring randomly chosen lattice site. If it is sent to a location inhabited by an older version, the new one is adopted. But, if it is transmitted to a place where an even newer version exists, the older word

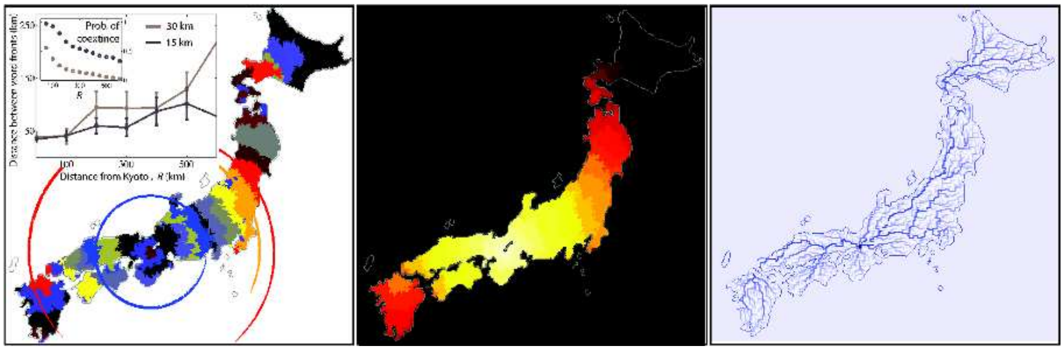


Figure 10: *Snapshots of a simulation showing the spatial dynamics of word spreading over the Japanese mainland. (left panel) Ongoing spreading where each colour represent a different word. Blue and red circles show two examples where the same word forms is found symmetrically on either side of kyoto. The graph in the upper left corner shows the mean distance between two adjacent fronts (averaged over many runs) as a function of distance from Kyoto. The orange broken circle belongs to a word which only is present at Kyoto's east side. The probability that a word coexists on both sides decay with distance away from Kyoto in a way shown in the inset. We investigated the behaviour when the spatial resolution (width of lattice site) was set to 15 and 30 km, respectively. (middle panel) Age landscape illustrating how older and older words (yellow to dark red colouring) are encountered as one moves further away from Kyoto. (right panel) River landscape showing the path the new words took as they moved away from Kyoto. In order to improve the quality of the figures, a lattice spacing of $\Delta = 5$ km was used.*

is ignored; new concepts always overrules old. We point out that our model captures the way in which new cultural traits invade new territories and not their coexistence. This means that new and old concepts in principle can live in parallel in the same region of space (it is, however, only the most recent that is transmitted) but we are interested in the most recent one. We have implemented our model in an interactive java applet, see cmol.nbi.dk

A snapshot of a simulation from our java applet is shown in Fig. 10. In the left panel each new word is given its own random colour and the source (Kyoto) is marked in black. The model gives rise to patterns of concentric rings penetrating the landscape moving outward from source. Also, the same colour can be found an either side of Kyoto without being present in the middle, a key feature of the real data (Fig.). If we calculate the corresponding radii for all colours in the landscape and average over many landscapes, the mean distance between two consecutive words increase when moving away from Kyoto as is shown in the graph in the left panel. This result obviously hangs on the frequency of new words f_{word} from Kyoto as well as the coarse graining of space. The graph therefore depicts two cases where we used lattice spacings of $\Delta = 15$ km and $\Delta = 30$ km where the word frequency was adjusted in each case such that about 20 words could be distinguished simultaneously on Hunshu island, as required by the data. Based on the fact that $v_{\mathrm{word}} = 1$ km/year we find from our model that new words are being coined in Kyoto on average every 30th year for $\Delta = 15$ km and every 60th year for $\Delta = 30$ km.

In addition to the geographical distribution of words, it is also interesting to see along which routes the words traveled. This can be quantified within our model if we at each lattice point store from which neighboring site the latest word came in addition to its age, similar to the river networks of the time walkers. By this we again follow information pointers downstream and eventually reach all the way back to the source, see Fig. 10. The river network is self-similar and obeys similar scaling relations as river networks of flowing water (but with different exponents).

- **Lesson 6:** Information spread while being sorted according to age: New is better than Old!

- **Lesson 7:** Building roads, or making "information highways", will homogenize the system. The roman empire got that right.

Questions:
1) Consider spreding of signals along a 1-d line, with new words appearing at position $x = 1$ with a high frequency (for example each time teach agent have been involved in one word exchange). At each step, select two neighbors, and let the youngest word spread to replace the oldest word. Plot number of words from position 1 to any position $x < N = 1000$ in a steady state situation.

Self-assembly of information in networks

To put our information spreading in a network perspective see Fig. 11, a network composed of individual agents, each of them connected to a number of acquaintances [52]. Each individual communicates with its immediate neighbors in order to exchange information about agents in other parts of the system.

When an acquaintance of agent number **5** in Fig. 11 obtains information about **5**, it sets its pointer to **5**, and the information starts aging. With successive communication events, the information spreads from agent to agent and gets older and older (we increase the age of all information (clocks in Fig. 11) when all links on average have participated in one communication event). When two agents compare the validity of their pointers to a target agent, like **1** and **6** to **5** in Fig. 11, they validate the newest information as

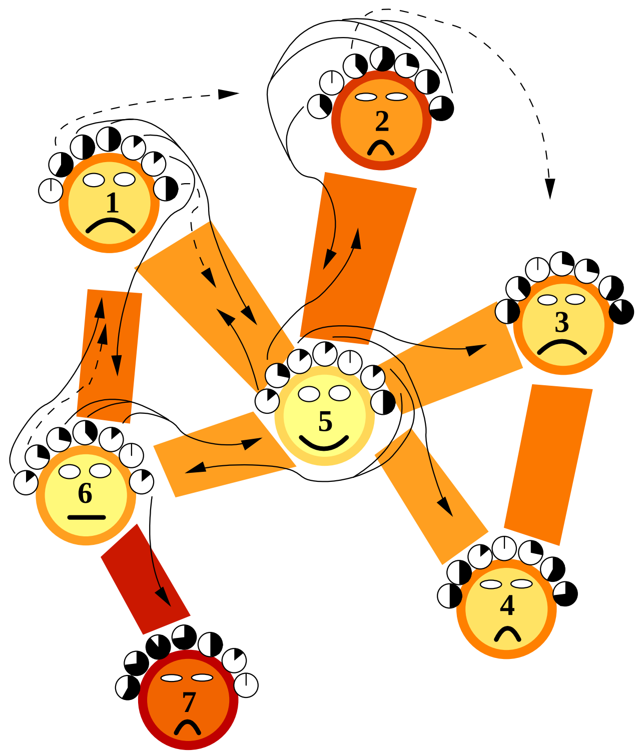


Figure 11: *Self-assembly of information as modeled in this paper. Agents at nodes communicate with their acquaintances about any third target agent in the network, and estimate the quality of the information by its age (clocks over the heads from left to right corresponds to age of information about agent 1, 2,... 7). The pointers are, for every agent, the acquaintance that connects most efficiently to each of the other agents in the system (dashed pointers are outdated).*

the most correct one. By letting the agents memorize the acquaintances that provided the newest information about other agents together with the age of this information, they will point in the direction of the fastest communication path from a target, which is typically close to the shortest path.

Figure 12 indicates the perception around the central node in a model network (see also Java applet. It is clear that the information is most up to date in the immediate neighborhood of the agent, but that distant communication-pathways extend the whole network.

- **Lesson 8:** Sorting information according to its age opens for robust routing on networks.

Self-organization of networks

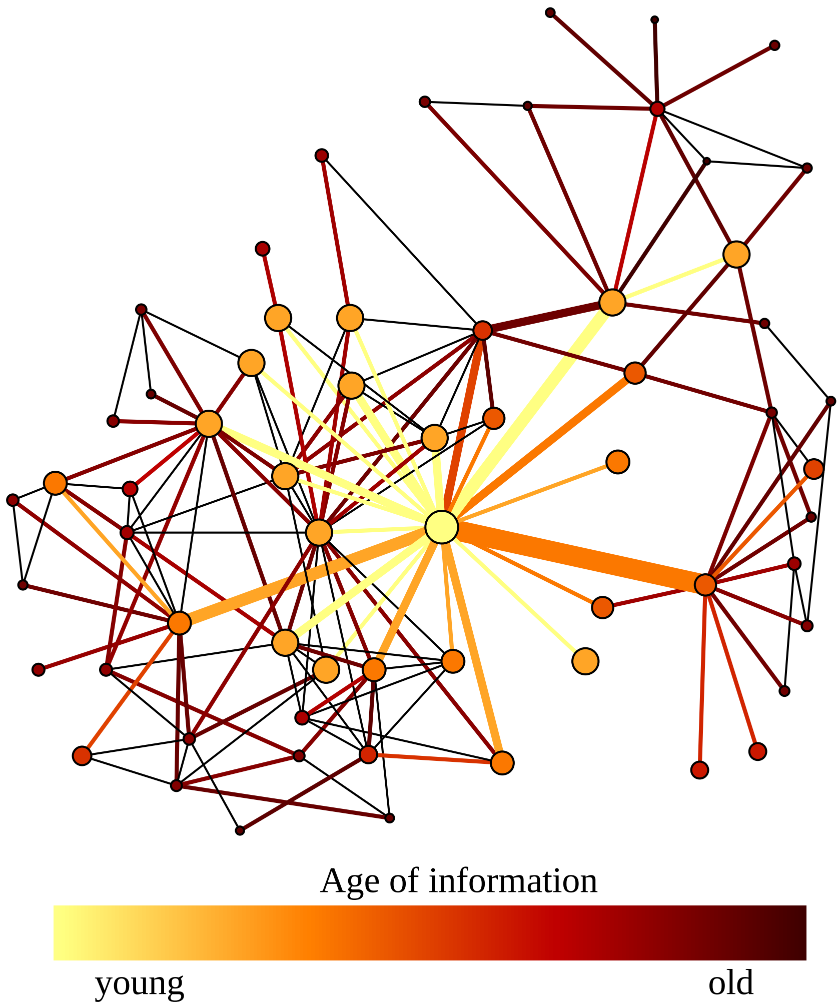


Figure 12: *The size and the color of the nodes reflect the age of the information the well connected agent in the middle has of other agents. The width of the links reflects the relative amount of information they transfer to this agent, and the color the average quality (age) of this information. It is clear that the agents make use of the hubs to create short communication-paths.*

Social mobility may be seen as the response to the quest for better information access in a social system. We let agents communicate to build a perception of a network like in the previous section, and further allow the agents to use this information to create strategic links. The core of such a simple self-organization model is formulated in the interplay between a Commuication and a rewiring step:

- *Communication:* Select a random link and let the two agents that it connects communicate about a random third agent. The two agents also update their information about each other.

- *Rewiring:* Select a random agent and let it use the local information to ask an acquaintance about whom to establish a link to, to shorten its distance to a randomly chosen other agent (the answer is the agent that the acquaintance points to). Subsequently a random agent loses one of its links.

The communication event is typically repeated of the order of number of links in the system for each rewiring event.

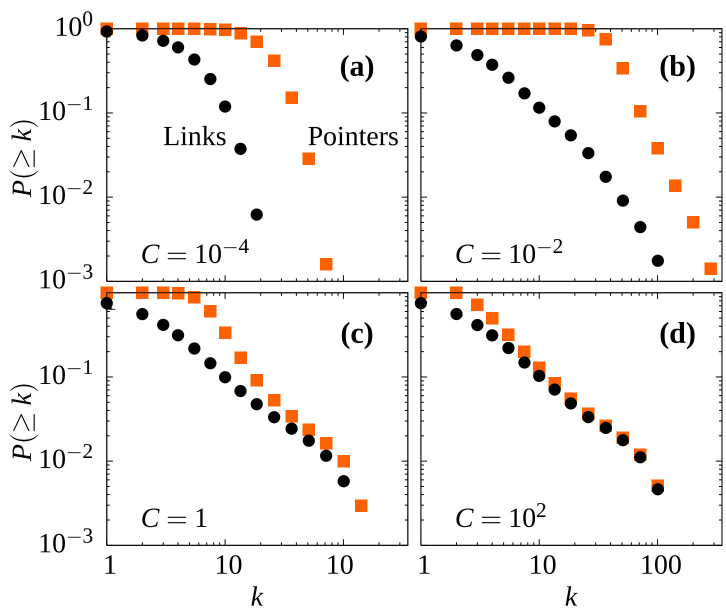


Figure 13: *Illustration of the feedback of communication on the topology of both the communication network and the perception network at 4 different levels of communication C. $C = 1$ corresponds to on average 1 communication event per link and rewiring event. The system size is $N = 1000$ agents connected by $L = 2500$ links. At lower level of communication, the degree distribution becomes random Erdos-Reney like, and at the sam time the links the agents believe they have (poinnters) deviates from the real links (links).*

To quantify th self-organization between communication, we in Fig. 13 show degree distributions Black dots) for simulations of a system with $N = 1000$ agents and $L = 2500$ links. Overall lesson is a gradual change from a narrow degree distribution at low C, to a broad degree distribution at high communication C.

Notice that the model does not give robust scale free behaviour, as the degree distribution very much depends on details of communication behaviour. If agents have limited capacity for responding, ta cutoff will for example develop, possibly with ways to bypass the then inefficient hups.

Question:
1) Simulate a simplistic version of the above model: At each time step select a node, and allow this node to make a link to the one of its neighbors neighbor who have the highest degree. If selected node have no link, let it make a link to a random node. In any case remove a random link. Do the simulation for N=100 nodes and 150 links starting with a random Erdos Reynei network.

- **Lesson 9:** Hunting for new information while networking reinforces social hubs. It "pays" to be name-dropping.

Social seggregation revisited: A network approach

Social groups with different tastes, political convictions, and religious beliefs emerge and disappear on all scales. But how do they form? Do they form because heterogeneous people search and navigate their social network to find like-minded people, or because interests are reinforced by interactions between people in social networks with modular topologies? However, if groups form because people are inherently different and search for people who are like them, then the question becomes where the different interests come from. If, instead, it is because interests are reinforced in modular social networks [51], then we must first understand why social networks are modular. Following ref. [52] we here combine the two views to propose that group formation can occur without assuming that people have different intrinsic properties.

We introduce a simple agent-based network model of communication and social navigation. We use social navigation to represent peoples' attempt to come nearer to the information source in the network they find interesting. The model is inspired by everyday human conversation and captures the feedback between interest formation and emergence of social structures. Taking this approach, we acknowledge that the goal of individuals to understand and agree with their closest associates [53, 54, 55] can be obtained either by adjusting their interests or by adjusting their contacts. Agents is asumed to have one goal: to be updated about topics they find interesting,with objects being limited to the agents themselves!

The model is defined in terms of N connected by a fixed number of links L, as its behaviour depend on communication and social navigation through 3 parameters: the communication to social navigation ratio C/R, the interest size η, and the flexibility μ. Central to the model is to build and use a perception of the system. We therefore give each agent i an individual memory

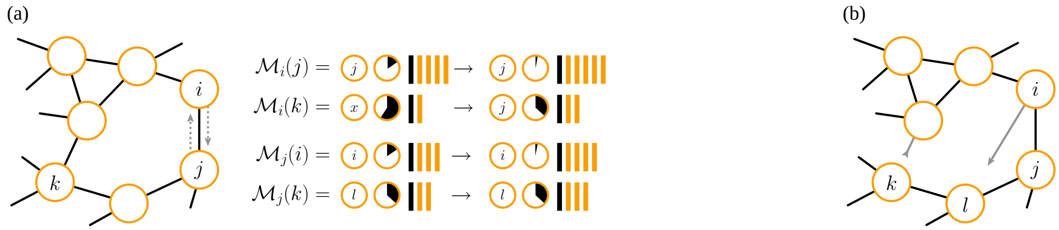


Figure 14: Modeling communication and social navigation. The depicted memory illustrates, from left to right: agent indices for memory, clocks for the quality memory, and bars for the interest memory. I.e. the number of bars in $\mathcal{M}_i(k)$ corresponds to the number of elements $m_i(k)$ of agent i's interest memory that are allocated to agent k, with the black bar representing the global and fixed interest. (a) *Communication C:* A random agent i selects one of her neighbors j proportional to her interest in j. Similarly, either of the two agents selects agent k from her interest memory. When agents i and j communicate, they update their interest memories[a] and the information about each other[b], and the agent with the oldest memory about k updates her information about k[c]. (b) *Social navigation R:* A random agent i selects an agent k proportional to her interest in k and recollects the friend $j = \mathcal{M}_i^{\text{rec}}(k)$ who provided her with information about k. Subsequently agent i forms a link to her friend's friend, that is j's friend $l = \mathcal{M}_j^{\text{rec}}(k)$, to shorten her distance to k. To keep the number of links fixed in the network, one random agent loses one random link (Footnotes refer to updates of the memory in the communication event: [a]Agents i and j replace a fraction μ of their interest memory with k. Similarly, both agents reciprocally increase their interest in the other agent. [b]Both agents update their recollection and quality memories: $\mathcal{M}_i^{\text{rec}}(j) = j$ and $\mathcal{M}_i^{\text{age}}(j) = 0$ for i, and $\mathcal{M}_j^{\text{rec}}(i) = i$ and $\mathcal{M}_j^{\text{age}}(i) = 0$ for j.).

$\mathcal{M}_i$. The memory consists of three one-dimensional arrays,

$$\mathcal{M}_i = \begin{cases} \mathcal{M}_i^{\text{rec}} & \text{a recollection of who provided the information} \\ \mathcal{M}_i^{\text{age}} & \text{the quality (age) of the information} \\ \mathcal{M}_i^{\text{int}} & \text{the interest preferences in agents} \end{cases}$$

The recollection memory contains N names of the friends $\mathcal{M}_i^{\text{rec}}(j)$ that provided information about agents $j = 1 \ldots N$. To compare the quality of the information with friends, the quality memory stores the age of each of the N pieces of information. Finally, the interest memory contains $\eta N \geq N$ names of agents in a proportion that reflects the interest in these agents. Recollection and quality memories $\mathcal{M}_i^{\text{rec}}$ and $\mathcal{M}_i^{\text{age}}$ constitute agent i's local map of the social structure [**?**], and $\mathcal{M}_i^{\text{int}}$ is the interest memory with priorities of other agents (see Fig.).

Also this network model is executed in time steps, each consisting of one of *Communication C* and *Social navigation R* where the selection of communication topic and social-navigation direction are associated with interests as described in Fig. : To select a topic of communication or direction of social navigation, an agent simply picks a random element in her interest memory and reads off the name of the agent that she has stored there.

Because the agents also update their interest memories when they communicate, there is a feedback between the organization and the agents' interests which makes the interest memory of crucial importance. The socia structure will shape itself to fit the interests of the agents, which in turn is determined by the social structure.

In paticular, Global interests generate the hierarchical organization described in previous chapter; local interests generate a heterogeneous organization. By letting the first N elements of the interest memory form the global interest and the remaining $\eta N - N$ elements form the local interest, the parameter η provides full control of the strength of the feedback. The elements of the static global interests are fixed to each of the N agents' names, whereas the elements of local interest are updated by communication.

For $\eta = 1$, any topic is selected with equal chance, whereas larger η increases the bias of proportionate local interest selection over random global interest selection. The modeling of proportional allocation of interests is not only the simplest possible mechanism; it is also in accord with H. Spencer's observation of proportionality between interest and previous experience [51]:

We initiate each simulation by filling the local interest memory with random names. Later, each turn agent i communicates with or about another agent j, the name of j randomly replaces a fraction μ of i's dynamic interest memory. That is, $\mathcal{M}_i^{\text{int}}(\alpha) \to j$ for $\mu(\eta N - N)$ values of $\alpha \in [N+1, \eta N]$. Thereby old priorities will fade as they are replaced by new topics of interest. We denote by $m_i(k)$ the number of elements of agent i's interest memory that are allocated to agent k. When selecting a communication topic or the direction of social navigation, agent i, by choosing a random element in her interest memory, selects agent k proportional to $m_i(k)$.

We increment the age $\mathcal{M}^{\text{age}}$ by one after every L communication events. Because every agent always has information with age 0 about itself, $\mathcal{M}_i^{\text{age}}(i) = 0$, the age of the information about an agent becomes older as, through communication, it percolates away from the agent in the network. When two agents communicate about a third agent, and evaluate the quality of the information based on its age, the agent with the newest information tends to be closer to the third agent. This guarantees that the recollection memory works as an efficient local map of the social structure.

Social navigation, which corresponds to a rewiring of the network, is a slow process compared to communication. If this were not the case, random people would share reliable information with anybody and the network become meaningless, i.e. random as seein in previous chapter. We therefore simulated the model with on average $C/R = 10$ communications per link for each rewiring event in the system. Moreover, because friends refer to the particular agents that have provided the most recent information about the selected agent, new links are formed on the basis of the memory rather than on the basis of the present network.

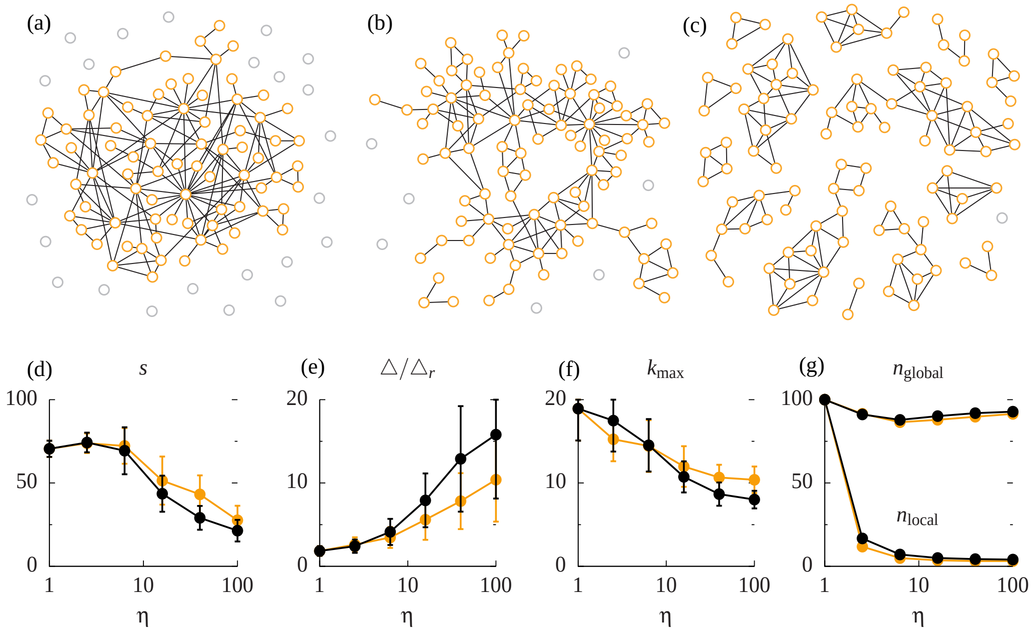


Figure 15: Local communication generates social groups. From left to right, the networks are generated with increasing interest size η, ($\eta = 1$ in (a), $\eta = 10$ in (b), and $\eta = 100$ in (c)). As a function of η, the bottom panels illustrate the typical module size in (d), the cliquishness in (e), the maximum degree in the network in (f), and social horizon in (g). Simulations are based on $C/R = 10$ communication events per link for each social navigation event in the system, with system size fixed to 100 agents and 150 links. The results are robust to a hundredfold drop in the communication to rewiring ratio, but break down at an even lower communication rate when only small groups can be maintained by the communication. In addition, panels (d-g) illustrate the dependence of the rate of interest adaptation, or flexibility, with a $\mu = 0.01\%$ change of the interest elements per communication event for stubborn adaptation (black lines), and a $\mu = 1\%$ change for flexible adaptation (shaded lines). Stubborn adaptation corresponds to a flexibility of 15 percent change in the interest memory when all links are changed once, whereas flexible adaptation corresponds to complete reallocation.

Most remarkably, the model generates interest groups and modular social networks without assuming that people are different from the beginning. The mechanism that drives the process is a feedback between interest formation and the emergence of social structures catalyzed by the flow of information. Figs. 15(a-c) show three networks generated by interest sizes $\eta = 1$, $\eta = 10$, and $\eta = 100$ respectively. That is, in the network in Fig. 15(a), there is only random global interest selection, whereas the more modular networks in Figs. 15(b) and 15(c) are generated with dominating local proportionate interest selection.

When close-by agents receive more attention, they will also be frequent targets of social navigation. As Fig. 15(e) illustrates, this strongly affects the abundance of triads, here measured in units of the random expectation of triangles $\triangle/\triangle_r$ [**?**]. When agents shift their attention to their neighborhood, the centralized network breaks down. Figure 15(f), showing the typical size of the largest hub, k_{max}, captures this transformation. Overall, for increasing but small interest size η, the largest hubs receive more attention, which allows the system to remain in one module. When η exceeds 5, s decreases strongly, the degree distribution narrows further and the number of triangular cliques increases substantially.

The transition from a centralized to a modular structure, driven by the potential to form individual interests, is of course also manifested in the interest memory itself. To quantify this gransition, the local social horizon,

$$n_{\text{local}} = \left\langle \frac{\eta N}{\langle m_i^2(j)\rangle / \langle m_i(j)\rangle} \right\rangle, \tag{13}$$

with a denominator, with averages over j, that corresponds to agent i's typical interest allocation in an agent. The typical number of individuals an agent has in her interest memory is simply the number of such allocations there is room for in an agent's interest memory, averaged over all agents. Because only a limited amount of information is exchanged with agents outside the local social horizon, it can also be thought of as an information horizon [56].

The global social horizon,

$$n_{\text{global}} = \frac{\eta N^2}{\langle m^2(j)\rangle / \langle m(j)\rangle}, \tag{14}$$

is calculated by pooling the agents' interest memories together into $m(j)$ for the total number of elements allocated to agent j. Figure 15(g) shows the local horizon of the individual agent together with the global horizon of all individuals. As η increases, n_{local} collapses while n_{global} remains on the order of N; the development toward social cliques is democratic, with anyone getting a fair share of attention while still allowing people to focus locally on members of their particular "club." For further details of this model, go to ref. [52].

- **Lesson 10:** Seggregation may develop among identical agents, as they self-organize into cliques that primarely are interested in themselves.

- **Lesson 11:** Social organization is hugely influence by communication rules/constraints. Adding global broadcasting to a social network will increase global avareness, and reduce clustering.

Questions:

1) Consider N=100 agents that each have a memory list with 10 slots recollecting the sequence of people they talked to the last to times. At each time step select an agent i, and select one peorson j in the agents memory list. Move the agent to the group that this person belong to, and replace one of the slots in the agents memory list with a memory of j. Furtehr with probability p, also let the agent talk to a random person, and move to thsi persons group. Start simulations with all agents seperate, and study steady state distribution of number of groups as well as sizes of groups.

→ more agent based models of social systems:

- **Thresholds & Bistable phenomena** [10]
- **Thresholds and a society of excitable agents** [63, 64]
- **Cellular automata:** [1, 3, 63]
- **Self organized criticality:** Grasbergers office version of the sandpile model [62, 63].
- **Diversity versus monoculture:** language diversity, naming game, voter models [65]
- **Game theory and Bluff:** Von Neumann Poker [60], Games for Squirrels and Generals.
- **Tragedy of the commons:** Rock-paper-scissor game [66], ...
- **Game theory and Models of cooperations:** Prisoners dilemma [16].
- **Nash equilibrium:** [61, 60]
- **Asymetric information:** [7]
- **Crash models & financial depth** [59]

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